TABLE III

Strip Width (mm)	ϵ_{eff} (st)	ϵ_{eff} (0)	$\epsilon_{eff}(w)$ at 4 GHz	8 GHz	12 GHz	16 GHz
Fused Quartz						
3.00	3.25	3.26	3.28	3.31	3.36	
1.50	3.06	3.09	3.10	3.12	3.15	
1.00	2.96	2.97	2.98	2.99	3.02	3.06
0.50	2.81	2.86	2.87	2.88	2.89	
0.15	2.63	2.66	2.67	2.67	2.68	
Alumina						
2.00	8.30	7.85	8.06	8.31	8.60	
0.90	7.63	7.07	7.27	7 40	7.60	
0.58	7.33	6.86	6.93	7 11	7.31	7.52
0.20	6.76	6.25	6.30	6.40	6.50	
0.07	6.41	6.00	6.00	6.10	6.20	

ment with the static theory (Fig. 4). The orientation of the crystallites depends on the manufacturing processes and is not necessarily constant over the substrate. The anisotropy in alumina substrates can be inconvenient, especially when used for circuits comprising narrow-band filters and when experimentally verifying theories.

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The Green's Function for Poisson's Equation in a Two-Dielectric Region

ANTONIO F. DOS SANTOS AND VICTOR R. VIEIRA

Abstract—**The validity of the reciprocity relation satisfied by the Green's function for Poisson's equation in a two-dielectric region is briefly discussed.**

INTRODUCTION

In calculating the parameters of a stripline by variational techniques it is often necessary to determine first a Green's function for the two-dimensional Poisson's equation in the region bounded by the two conductors [1], [2]. Contrary to the case of a single dielectric [2], the symmetry properties of the Green's function in a two-dielectric region do not appear to have been dealt with in the literature.

The aim of this short paper is to point out that the reciprocity relation satisfied by the Green's function in the latter case is only valid for a specific form of the right-hand side of the differential equation defining the Green's function. Only the Green's function subject to Dirichlet boundary conditions will be considered.

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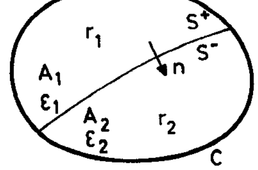

Fig. 1.

RECIPROCITY RELATION

Let $G(r, r_0)$ be the function satisfying the following conditions in the two-dielectric region $A = A_1 \cup A_2$ (see Fig. 1):

$$\nabla^2 G = -\frac{1}{\epsilon}\delta(r - r_0), \qquad r \in A \tag{1a}$$

$$G = 0, \qquad r \in C \tag{1b}$$

$$G|_{S^+} = G|_{S^-} \tag{1c}$$

$$\epsilon_1 \frac{\partial G}{\partial n}\bigg|_{S^+} = \epsilon_2 \frac{\partial G}{\partial n}\bigg|_{S^-} \tag{1d}$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y_2}.$$

Applying the Green's identity [2] separately to regions A_1 and A_2, in which G and its first-order partial derivatives are continuous with the only exception of the source point ($r = r_0$), the following equations are readily obtained:

$$\frac{1}{\epsilon_1} G_2|_{r=r_1} = \int_S \left(G_1 \frac{\partial G_2}{\partial n} - G_2 \frac{\partial G_1}{\partial n}\right)_{r \in S^+} dl \tag{2a}$$

$$-\frac{1}{\epsilon_2} G_1|_{r=r_2} = -\int_S \left(G_1 \frac{\partial G_2}{\partial n} - G_2 \frac{\partial G_1}{\partial n}\right)_{r \in S^-} dl \tag{2b}$$

where G_1 and G_2 denote $G(r, r_1)$ and $G(r, r_2)$, respectively. Substitution of (1c) and (1d) into these equations yields the reciprocity relation

$$G(r_1, r_2) = G(r_2, r_1) \tag{3}$$

which shows that the Green's function is symmetric in its two arguments. Examination of (2a) and (2b) shows, however, that if the RHS of (1a) is simply $\delta(r - r_0)$, the reciprocity relation (3) no longer holds. In fact, it can be shown without much difficulty that in this case, the function G is not the true Green's function for Poisson's equation subject to the boundary conditions (1c) and (1d).

Finally we note that in view of relation (3), to determine G completely it is sufficient to consider the case where the source point is located in one of the two-dielectric regions, e.g., A_1.

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"Unfolding" the Lange Coupler

RAYMOND WAUGH AND DAVID LACOMBE

The broad-band microstrip quadrature coupler described by Lange [1] is shown in Fig. 1(a). True quadrature coupling over an octave is realized as a consequence of the interdigital coupling section which compensates for even- and odd-mode phase velocity dispersion over the wide frequency range. A power-split variation between the direct and coupled ports, ports 3 and 4, respectively, in Fig. 1(a), of

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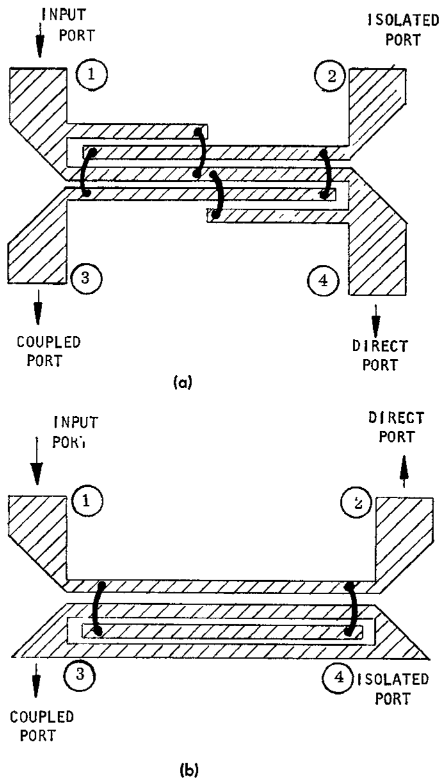

Fig. 1. (a) Lange coupler. (b) "Unfolded" Lange coupler.

less than 0.5 dB is typical with high isolation at port 2 and low-input VSWR at port 1. This short paper shows how the Lange coupler can be "unfolded," that is, interchanging the roles of the direct and isolated ports as illustrated in Fig. 1(b). Port 2 now becomes the direct port and port 4 the isolated port. One-half of the normal coupling section is simply turned over to produce, effectively, a broad-band backward-wave quadrature coupler. The ability to choose either port 2 or 4 as the transmitted port offers the MIC circuit designer considerable flexibility when routing microwave signals in sophisticated subsystems where real-estate minimization is important.

Measured performance of a regular C-band Lange coupler and that of its "unfolded" complement is presented in Figs. 2 and 3, respectively. Both circuits were fabricated on 0.025-in thick 99.6-percent alumina having a surface finish of 10 μin and metallized with chrome/gold. Table I summarizes the data, indicating that either structure yields similar results. Over slightly greater than an octave, a mean power split of 3.35 dB was achieved with a maximum of 0.4-dB insertion loss, maximum VSWR of 1.32, and a minimum isolation of 17.5 dB. The maximum deviation from mean power split was ± 0.5 dB for both circuits and deviation from true 90° quadrature did not exceed 8.5°. The flat 90° relative phase shift truly verifies the even/odd mode phase equalization concept. Both variations of the coupler had a 0.190-in long coupling section with uniform 0.002-in gapwidths and 0.0028-in linewidths along the coupling length. It is noteworthy here to indicate that the design gapwidth was 0.001 in; the extra 0.001-in results from undercutting, commonly referred to as etch factor, being approximately 0.0005 in for each line bordering the gap. Backplating (gold electroplating) after final etch will reduce the gapwidth but experience indicates that no significant coupling tightness warrants the additional processing.

Both types of Lange couplers are readily reproduced in S and X band by scaling the coupling region length, nominally a quarter wavelength at midband. Varying linewidth and gapwidth about the values presented above alters mainly the degree of coupling. Scaling high in frequency has been accomplished to cover the 8- to 18-GHz [2] range by altering only the coupling section length; however, care must be exercised in preserving gapwidth and linewidth uniformity so as to retain a reasonable power split. In Ku band, the ceramic microstrip designer must accept increased insertion loss and launcher discontinuity problems or revert to a more controlled medium such as stripline for wide-band quadrature hybrid performance.

It is anticipated that this "unfolding" idea can be extended to de-

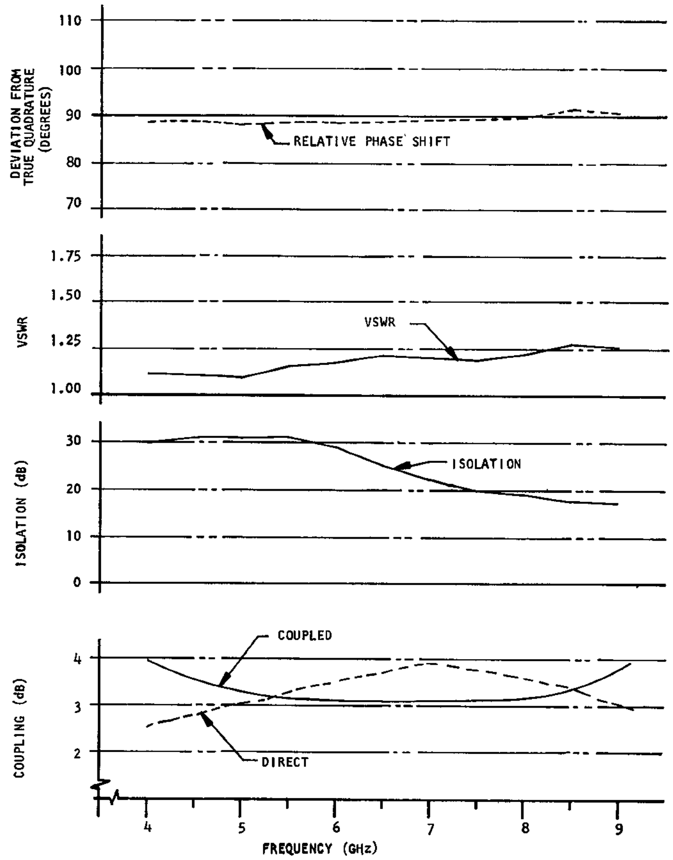

Fig. 2. Measured performance of a C-band Lange coupler.

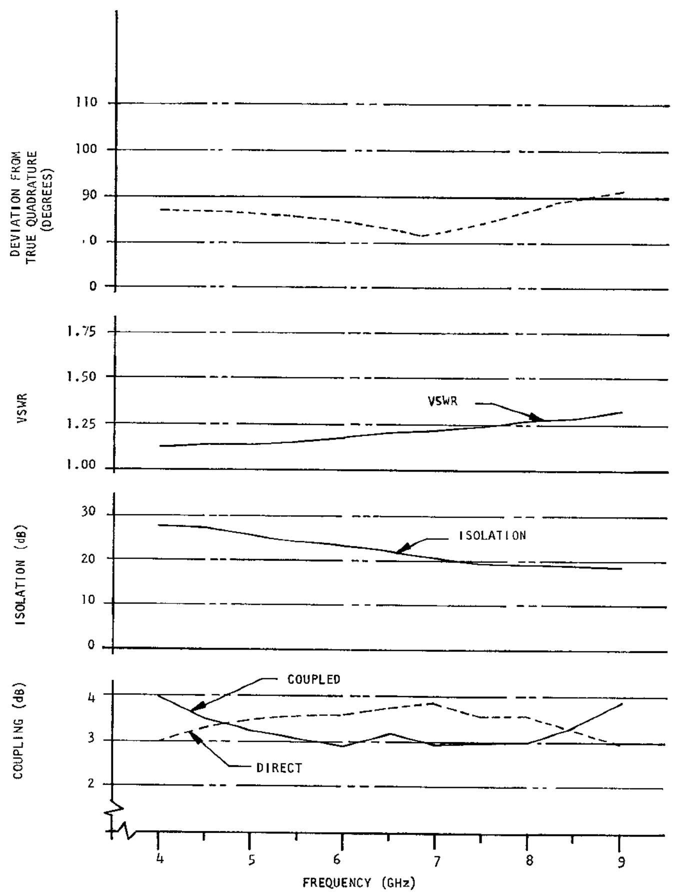

Fig. 3. Measured performance of a C-band "unfolded" Lange coupler.

TABLE I
SUMMARY OF MEASURED DATA FOR A C-BAND LANGE COUPLER AND ITS "UNFOLDED" COMPLEMENT

	BANDWIDTH (%)	MEAN POWER SPLIT (dB)	MAX. DEVIATION FROM MEAN POWER SPLIT (dB)	MAX. INSERTION LOSS (dB)	MIN. ISOLATION (dB)	MAX. VSWR	MAX. DEVIATION FROM TRUE QUADRATURE
REGULAR LANGE COUPLER	73	3.43	± .5	.4	18	1.32	8.5°
"UNFOLDED" LANGE COUPLER	68	3.3	± .5	.32	17.5	1.27	± 2°

velop a family of broad-band backward-wave microstrip couplers with coupling values other than 3 dB. The significant practical advantage when designing on 0.025-in ceramic microstrip may be the elimination of ultranarrow gapwidths as required, for example, by Podell's "wiggly"-line backward-wave coupler [3] for coupling values typically under 7 dB. This extension, however, has not been pursued.

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Dissipation and Scattering Matrices of Lossy Junctions

J. HELSZAJN

***Abstract*—The purpose of this short paper is to construct the dissipation and scattering matrices of lossy junctions in terms of the eigenvalues of the dissipation matrix. This removes the need to rely on inequality relations between the scattering parameters of lossy circulators. The eigenvalues of the dissipation, scattering, and admittance matrices are related. The eigenvalues of the dissipation matrix give the dissipation associated with each possible way of exciting the junction. The ones of the scattering matrix give the reflection coefficients associated with these different excitations. The admittance eigenvalues define in each instance the eigennetworks of the junction. This leads to the definition of the entries of the dissipation matrix in terms of the loaded and unloaded Q-factors of the junction eigennetworks. The scattering matrices of a number of lossy 3-port junctions are also constructed directly in terms of the elements of the eigennetworks.**

I. INTRODUCTION

The general relation between the coefficients of the scattering matrix for a lossy symmetrical 3-port circulator has been discussed by a number of authors [1]–[4]. The insertion loss has also been derived in the case of the lumped-element circulator [5]. The most general relation between the scattering coefficients has been given graphically [3] in terms of the dissipation matrix. Inequality relations for semi-ideal circulators in which the insertion loss is not zero and either the isolation or VSWR is ideal have also been discussed [4].

The purpose of this short paper is to directly construct the dissipation and scattering matrices of lossy junctions in terms of the eigenvalues of the dissipation matrix. This removes the need to rely on inequality relations between the scattering parameters of lossy circulators. The scattering matrix of lossy circulators is also constructed directly in terms of the elements of the eigennetworks.

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The short paper starts by relating the eigenvalues of the dissipation, scattering, and admittance matrices. The eigenvalues of the dissipation matrix give the dissipation associated with each possible way of exciting the junction. The ones of the scattering matrix give the reflection coefficients associated with these different excitations. The admittance eigenvalues define in each instance the eigennetworks of the junction. This leads to the definition of the entries of the dissipation matrix in terms of the loaded and unloaded Q-factors of the junction eigennetworks. In the most usual arrangement, one of the eigenvalues is associated with a nonresonant shunt network that appears as a short circuit at the reference terminals of the junction, and is therefore always unity. The other eigenvalues are the reflection coefficients of resonant shunt networks, and the presence of loss means that the magnitudes of these eigenvalues will depart from unity. The amplitudes of the eigenvalues are, in general, unequal.

The theory is applied to reciprocal 3-port junctions, to 3-port junction circulators, and to semi-ideal 3-port circulators. It may also be applied to the scattering matrices associated with the different stages in the adjustment of the circulators described elsewhere [7], [11], [12].

II. EIGENVALUES OF SCATTERING AND DISSIPATION MATRICES

For a lossy circulator, the dissipation matrix Q must be positive real [3], [6]:

$$Q = I - (S^*)^T(S) \tag{1}$$

where I is a unit matrix and S is the scattering matrix.

In the case of a symmetrical 3-port junction, one has for the S matrix

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{13} & S_{11} & S_{12} \\ S_{12} & S_{13} & S_{11} \end{bmatrix}. \tag{2}$$

The matrix Q is given by [3], [4], [6]

$$Q = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{13} & q_{11} & q_{12} \\ q_{12} & q_{13} & q_{11} \end{bmatrix} \tag{3}$$

where

$$q_{11} = 1 - |S_{11}|^2 - |S_{12}|^2 - |S_{13}|^2 \tag{4}$$

$$q_{12} = S_{11}S_{12}^* + S_{12}S_{13}^* + S_{13}S_{11}^* \tag{5}$$

$$q_{13} = q_{12}^*. \tag{6}$$

The matrix Q is positive real provided

$$|q_{12}| < q_{11}. \tag{7}$$

The bounds of (7) are given in [3] with q_{11} and $|q_{12}|$ as parameters. This gives the allowable values of the scattering parameters.

In what follows, the above inequality relation between the scattering parameters will be replaced by implicit ones. This is done by directly forming the S- and Q-matrices of the junction in terms of eigenvalues of the dissipation matrix. These eigenvalues represent the dissipation associated with each possible way of exciting the junction. They are related to the loaded and unloaded Q-factors of the junction eigennetworks.

If the scattering and dissipation matrices have common eigenvectors, their eigenvalues may be related by the following theorem